Subspace learning with frequency regularizer: its application to face recognition

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Abstract

Subspace learning is an important technique to enhance the discriminative ability of feature representation and reduce the dimension to improve its efficiency. Due to limited training samples and the usual high-dimensional feature, subspace learning always suffers from overfitting problem, which affects its generalization performance. One possible method is to introduce prior information as a regularizer to constrain its solution space. Traditional regularizers are usually designed in spatial domain, which usually make the projection smooth. In this work, we propose a frequency regularizer (FR), which suppresses the high frequency energy so that the smooth priori is incorporated. Two representative supervised subspace methods with frequency regularizer, FR-LDA and FR-SR are introduced and further applied to face recognition problem. Extensive experiments on popular face databases validate the effectiveness and superiority of FR based subspace learning compared to traditional subspace learning methods.

1. Introduction

Face recognition performance has been greatly improved in recent years, especially with the development of deep learning [20, 19]. High-dimensional face representation, followed by dimension reduction is one of the state-of-the-art methods in face recognition field [4, 18]. In this work, we mainly focus on the dimension reduction method, in particular subspace learning to improve its discriminative ability and generalization performance.

Subspace learning tries to seek a subspace that well classifies samples from the original high-dimensional data. In early times, a series of linear methods are proposed, in which PCA [22] and LDA [1] are two representative ones. In order to address the nonlinearity of data distribution, many nonlinear versions like kernel based subspace learning [26, 14] and manifold learning [21, 16] are also proposed. With the development of subspace learning in past decades, LDA related methods are still considered as one of the state-of-the-art methods due to its robust face recognition performance and computational efficiency [18, 10]. In the following, we make a simple review on LDA related subspace learning. For more comprehensive survey, please refer to [13].

LDA utilizes the Fisher criterion to seek a linear transformation by maximizing the ratio of between class and within class variations. In real applications, especially image related problems, LDA usually suffers from singularity problem because of the high-dimensional data and limited training sample size, and the optimal solution of LDA cannot be found directly. To address this problem, Fisher LDA [1] proposes to apply PCA on original data first to make within class scatter nonsingular and then LDA is further conducted on the reduced PCA subspace. Null space LDA [5] maximizes the between class scatter on the null space of within class scatter, so that the ratio of between and within class scatters are maximized. Direct LDA [27] directly maximizes the between class scatter while normalizes the within class scatter.

For image related problems, to apply the subspace learning, the images are usually firstly converted into vectors, which increases the dimension of data and drops the structure information of images. To address this problem, researchers further propose 2D based subspace learning methods like 2DPCA [25] and 2DLDA [12]. In 2D subspace learning, the scatter matrix is computed based on the image matrix rather than vector, so that that the dimension of data is greatly reduced and the singularity problem in subspace learning is avoided. Lei et al. [9] propose to introduce contextual constraint based LDA. The principal is that the projection coefficients in local neighborhood should be similar, so that the contextual (structure) information is introduced. The nonlinear version is further proposed in [11].

Yan et al. [24] unify almost all the subspace learning into graph embedding framework. By constructing different graphics, different subspace learning methods can be realized. With the formulation of graph embedding, the solu-

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tion of LDA can also be considered as a regression problem, which generates spectral regression based discriminative subspace learning methods [3].

Among many subspace learning variants, regularized subspace learning is one of the state-of-the-art methods because of its good performance and reliability in different cases, and it is widely applied to many works [10, 4]. Regularized LDA (RLDA) [7] add a matrix ($\epsilon\mathbf{I}$) to within class scatter to make the within class scatter nonsingular, so that the LDA is solvable. Sparse LDA [2, 28] introduces $L1$-norm or $L0$-norm to regularize the solution to finish the subspace learning and feature selection simultaneously, and its generalization is improved, especially when the training data size is small.

Most subspace learning methods characterize the smoothness of a solution in spatial domain. It is intuitive to evaluate this property in frequency domain, which is the main motivation of this work. We propose a novel frequency regularizer that minimizes the energy of high components in frequency domain and incorporate it with LDA and spectral regression. Experiments on various face databases validates the effectiveness of the frequency regularizer.

The remainder of this paper is organized as follows. Section 2 describes the frequency regularizer. Section 3 introduces two subspace learning methods incorporated with frequency regularizer. Experiments on PIE, Multi-PIE, FRGC and LFW face databases are illustrated in Section 4 and in Section 5, we conclude the paper.

2. Frequency regularizer

The most classic regularizer used in subspace learning is Tikhonov regularizer [7], which minimizes the $L2$-norm of projections to make the solution smooth. Later, $L0$-norm and $L1$-norm [2, 28] are introduced to make the solution sparse, which can be better interpreted. Recently, Lei and Li [9] design contextual regularizer to utilize contextual information to improve the generalization performance of subspace learning.

Most of the existing regularization methods are considered in the spatial domain. That is, specific constraints are introduced to the feature values in different dimensions. For example, contextual constrained LDA (CCLDA) [9] requires that the difference of projection coefficients in neighborhood is small. In this work, we propose a novel regularizer that characterises the property of solution in frequency domain. Considering the smoothness priori, it is intuitive to realize it by making the high frequency energy small in frequency domain. Taking discrete cosine transform (DCT) as an example, given a solution vector $w = [w_0, w_1, \cdots, w_{d-1}]^T \in \mathbb{R}^d$, its DCT resulting vector $u = [u_0, u_1, \cdots, u_{d-1}]^T$ can be computed as

$$u_j = \sum_{i=0}^{d-1} w_i cos(\frac{(2i+1)j\pi}{2d}), j = 0, 1, \cdots, d-1 \quad (1)$$

Denoting $F = [f_0, f_1, \cdots, f_{d-1}]$, where $f_j = [cos(\frac{j\pi}{2d}), cos(\frac{3j\pi}{2d}), \cdots, cos(\frac{(2d-1)j\pi}{2d})]^T$, the DCT on a vector w can be formulated in a matrix form of $u = F^T w$. Taking the top-k frequency components into account, the norm of top-k frequency components E_k can be computed as $E_k = trace(F_k^T w w^T F_k) = trace(w^T F_k F_k^T w)$, where $F_k = [f_{d-k}, f_{d-k+1}, \cdots, f_{d-1}]$. We call E_k a frequency regularizer (FR) in this work.

3. Subspace learning with frequency regularizer

3.1. FR-LDA

LDA tries to seek a subspace that minimizes the intra-differences between the samples from the same class, while maximizes the inter-differences between the samples from different classes, so that the discriminative ability is improved. Denoting the samples from the i-th class as $X_i = [x_{i1}, x_{i2}, \cdots, x_{iN_i}]$, where N_i is the number of sample from class i, the purpose of LDA is to maximize the ratio of between class and within class scatters, formulated as

$$w = \arg\max_w \frac{|w^T S_b w|}{|w^T S_w w|} \quad (2)$$

where the between scatter matrix S_b and within scatter matrix S_w are defined as

$$\begin{aligned} S_b &= \sum_{i=1}^{L} \frac{N_i}{N} (m - m_i)(m - m_i)^T \\ S_w &= \sum_{i=1}^{L} \sum_{j=1}^{N_i} \frac{1}{N} (x_{ij} - m_i)(x_{ij} - m_i)^T \end{aligned} \quad (3)$$

where N is the number of total samples and L is the number of sample class; m_i and m are the mean vector of class i and the whole sample set, respectively. By introducing frequency regularizer, the objective of FR-LDA can be formulated as

$$w = \arg\max_w \frac{|w^T S_b w|}{|w^T S_w w + \eta(w^T F_k F_k^T w)|} \quad (4)$$

The solution to FR-LDA can be obtained by the following steps.

1. whiten the matrix $S_w + \eta F_k F_k^T$ and preserve projections w_1 that make transformed matrix $w_1^T S_w w_1 + \eta(w_1^T F_k F_k^T w_1)$ nonsingular.

2. apply PCA to $w_1^T S_b w_1$ to derive $L-1$ projections w_2 corresponding to the non-zero eigenvalues.

3. The solution to Eq. 4 can be obtained by $w = w_1 w_2$.

3.2. FR spectral regression

Spectral regression [3] learns the subspace by two steps: (1) get the low dimensional-embedding in subspace; (2) learn the mapping between the source data and the low-dimensional embedding. With LDA objective, the low-dimensional embedding in subspace can be constructed directly as follows. First, we construct c vectors as

$$y_t = [\underbrace{0,\ldots,0}_{\sum_{i=1}^{t-1} N_i}, \underbrace{1,\ldots,1}_{N_t}, \underbrace{0,\ldots,0}_{\sum_{i=t+1}^{c} N_i}]; t = 1,\ldots,c \tag{5}$$

where c is the number of classes and N_i is the number of samples from i-th class. Second, we apply Gram-Shmidit method to orthogonalize it to derive $c-1$ informative vectors $Y = [y_1^T, y_2^T, \cdots, y_{L-1}^T]$. The remaining task of spectral regression is to learn a mapping between source data set X and low-dimensional embedding Y. Under linear assumption, the regression problem can be formulated as $Y = X^T W$, where W is the projections. By imposing frequency regularizer, the objective of spectral regression can be formulated as

$$W = \arg\min_{W} ||Y - X^T W||^2 + \eta \cdot trace(W^T F_k F_k^T W) \tag{6}$$

By setting the derivatives of objective function with respect to W zero, the solution to Eq. 6 can be derived as $W = (XX^T + F_k F_k^T)^{-1} XY$.

4. Experiments

In this part, we apply subspace learning to face recognition to evaluate the effect of frequency regularizer. Both the traditional constrained face databases like PIE [17], Multi-PIE [6], FRGC [15] and unconstrained face database like LFW [8] are adopted. In each experiment, we randomly select N images from each subject to form the training/gallery set and the rest to form the probe set. The split is conducted 10 times and the mean face recognition accuracy over these 10 trials are reported. All the face images are cropped to 32×32 size according to the provided eye coordinates (for PIE, Multi-PIE and FRGC) or aligned images (for LFW) and the image intensity is scaled to $[0, 1]$. Fig. 1 shows some cropped examples from these four face databases.

4.1. Database Description

4.1.1 PIE

The PIE database consists of $41,468$ face images from 68 subjects under different poses, illuminations and expressions. In this experiment, Images under five near frontal poses (C05, C07, C09, C27 and C29) with all illumination and expression configurations are adopted. There are in total 170 images for each subject.

(a) PIE

(b) Multi-PIE

(c) FRGC

(d) LFW

Figure 1. Sample examples from (a) PIE, (b) Multi-PIE, (c) FRGC and (d) LFW databases.

4.1.2 Multi-PIE

The Multi-PIE database is an extended version of PIE. It contains $754,204$ images from 337 subjects with 15 poses and 20 illuminations, captured in four sessions during different periods. In this experiments, all the images with the frontal view and neutral expression under different illuminations are selected. There are in total $18,420$ images with 20-60 images per subject.

4.1.3 FRGC

FRGC was collected by University of Notre Dame, which is one of the largest face databases. Both controlled (indoor) and uncontrolled (outdoor) face images are collected. There are in total $36,818$ images from 535 subjects, including 22,387 controlled images and 14,431 uncontrolled images. In this experiment, we only use the images collected in outdoor environment to examine the performance to illumination variations. We select subjects that contain more than 10 uncontrolled face images. There are $13,901$ from 415 identities in this experiment.

4.1.4 LFW

LFW is a database collected from Internet to examine the face recognition performance in unconstrained scenarios. There are large expression, pose, occlusion variations. In the experiment, we select 143 subjects with more than 10 images per subject. The aligned version LFWa [23] is adopted.

4.2. Parameters clarification

There are mainly two parameters in FR related methods, i.e., η and k, which denotes how many high frequency components are selected. In the following, we examine the effect of these two parameters on FR-LDA when the gallery number is set to 5 on PIE database. We firstly set $k = 200$ and examine the performance with respect to different η from $\{1e-6, 1e-5, 1e-4, 1e-3, 1e-2, 0.1, 1, 10, 1e3, 1e4, 1e5, 1e6\}$. Fig. 2 (a) shows the corresponding performance curve. It is shown that the highest face recognition accuracy is achieved when $\eta = 1e4$. Next, we fix η to $1e4$, and evaluate the recognition performance corresponding to different values of k from {10, 20, 30, 40, 50, 60, 70, 80, 90, 100, 150, 200, 250, 300, 350, 400, 450, 500}, shown in Fig. 2 (b). The best face recognition performance is achieved when $k = 60$. Therefore, in all the following experiments, we set η and k to $1e4$ and 60, respectively.

4.3. Results and Discussions

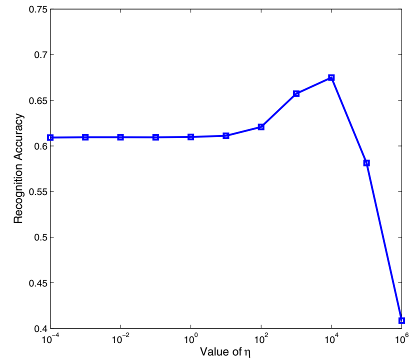
Tables 1 to 4 list the face recognition performance of different methods on four face databases. gN means N samples from each subjects are selected to form the gallery set. From the results, one can see that

1. Supervised methods like FLDA, 2DLDA, SR usually achieves significant better face recognition performance than unsupervised ones like PCA and 2DPCA.

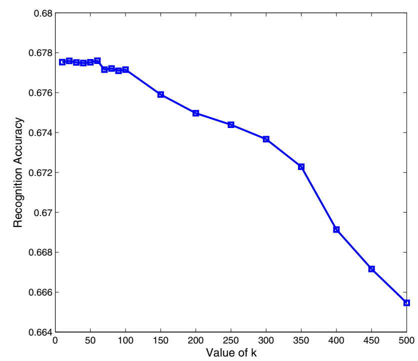
2. When the training sample is small, the performance of 2D matrix based subspace learning is usually higher than that of 1D vector based one, indicating the effectiveness of 2D matrix subspace learning to deal with small sample size and improve the generalization performance.

3. Incorporated with frequency regularizer, supervised subspace learning (i.e., FR-LDA and FR-SR) achieves higher face recognition rate than original subspace learning methods. On average, FR-LDA and FR-SR improve the performance of FLDA and SR by about 9.5% and 13%, respectively. FR-LDA even enhances the performance of 2DLDA by about 6.5%. The limited experimental results show that frequency regularizer is able to improve the recognition performance of subspace learning and the preliminary results are promising.

4. Compared to the constrained face recognition databases, the performance of unconstrained face recognition which is affected by pose, expression and occlusion simultaneously is greatly declined. For unconstrained face recognition, more robust and discriminative face representation and effective face pre-processing like pose normalization is desired to improve its performance further.



(a) Recognition accuracy with respect to different values of η.



(b) Recognition accuracy with respect to different values of k.

Figure 2. Face recognition accuracy of FR-LDA with respect to different values of η and k on PIE database.

5. Conclusions

This paper proposes a novel regularizer characterizing smoothness property in frequency domain. The frequency regularizer (FR) is incorporated with subspace learning and twp novel subspace learning methods, namely FR-LDA and FR-SR are presented. The performance of these two methods is investigated on different face databases, compared with traditional subspace learning methods including vector and matrix based formulations. The better face recognition accuracy validate that FR is an effective regularizer and

Table 1. Face recognition accuracy (mean±std.) on PIE database.

Methods	g3	g4	g5	g6	g10	g20
PCA	16.16±0.56	18.83±0.45	21.84±0.47	24.72±0.73	33.47±0.59	49.36±0.87
2DPCA	15.27±0.53	17.68±0.42	20.54±0.39	23.08±0.71	31.03±0.64	45.73±0.92
FLDA	43.97±1.47	50.91±2.25	59.36±2.01	62.49±1.84	70.04±0.92	78.01±1.07
2DLDA	49.92±1.42	55.40±1.66	61.09±1.31	64.45±1.33	73.90±1.02	84.77±0.71
SR	52.14±1.17	57.79±1.49	61.70±1.45	64.84±1.18	67.14±0.88	77.90±0.95
FR-LDA	54.29±1.69	61.73±1.83	67.76±1.17	72.45±1.15	81.98±0.95	90.95±0.70
FR-SR	50.00±1.80	58.12±2.02	64.53±1.25	69.71±1.21	80.61±0.98	90.54±0.76

Table 2. Face recognition accuracy (mean±std.) on Multi-PIE database.

Methods	g3	g4	g5	g6	g10	g20
PCA	18.55±0.34	22.94±0.23	26.57±0.26	30.13±0.37	45.56±0.24	64.31±0.48
2DPCA	16.85±0.34	20.83±0.21	24.28±0.29	27.42±0.29	42.36±0.27	60.76±0.56
FLDA	58.74±0.73	62.62±1.29	66.90±0.77	74.52±1.07	87.57±0.56	95.22±0.27
2DLDA	66.25±1.03	71.78±0.58	76.25±0.38	79.10±0.82	87.89±0.37	94.21±0.26
SR	45.47±0.92	50.78±1.33	59.75±0.86	65.51±1.45	82.44±0.83	93.06±0.46
FR-LDA	69.83±0.58	76.01±0.80	80.37±0.60	83.13±0.74	92.29±0.33	96.99±0.24
FR-SR	68.03±0.65	76.19±0.87	81.54±0.63	84.74±0.70	93.01±0.30	97.37±0.28

Table 3. Face recognition accuracy (mean±std.) on FRGC database.

Methods	g3	g4	g5	g6	g10	g20
PCA	33.52±0.43	38.76±0.40	42.78±0.63	46.54±0.29	55.99±0.74	70.13±0.73
2DPCA	31.29±0.37	35.97±0.49	39.94±0.63	43.55±0.37	52.24±0.69	66.59±0.63
FLDA	51.75±0.77	57.67±1.16	70.20±0.74	75.24±0.81	91.87±0.80	94.14±0.31
2DLDA	57.51±0.83	63.61±0.88	67.79±0.78	71.27±0.99	83.40±1.25	90.08±0.40
SR	38.45±0.87	54.64±1.08	61.69±1.22	66.85±1.39	86.52±0.93	93.11±0.52
FR-LDA	65.67±0.64	71.71±0.73	75.70±0.80	78.66±0.41	94.79±0.51	97.20±0.17
FR-SR	63.05±0.53	71.13±0.79	75.90±0.58	79.58±0.50	94.13±0.74	96.92±0.20

Table 4. Face recognition accuracy (mean±std.) on LFW database.

Methods	g3	g4	g5	g6	g7	g8
PCA	8.38±0.69	9.26±0.42	10.31±0.81	10.81±0.63	11.62±0.50	12.55±0.59
2DPCA	7.80±0.64	8.56±0.50	9.53±0.77	9.88±0.82	10.49±0.57	11.51±0.66
FLDA	22.88±2.03	24.19±2.17	25.09±2.24	25.65±1.44	28.89±1.67	31.36±2.03
2DLDA	22.95±1.58	26.42±2.07	29.93±1.24	32.29±1.26	34.51±1.33	35.77±1.77
SR	18.88±1.46	19.29±1.61	20.76±1.60	21.63±1.09	25.22±1.78	26.98±1.89
FR-LDA	27.76±2.15	33.96±3.45	36.74±2.98	38.91±2.25	43.13±2.30	44.53±2.35
FR-SR	26.06±2.14	33.12±3.31	37.21±2.81	40.39±2.43	45.57±2.01	47.36±2.60

has potential to be incorporated with other machine learning methods. In the future, we will take more experiments in different cases to analyze the performance of FR comprehensively.

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